Chasing the Hyper-Sphere in Real Space Geometry

Real space geometry has its roots in the work of Dr. S. K. Kapoor, and his knowledge of Vedic Mathematics, the Vedic literature and modern mathematics. I first came across his work through my interest in a paper he had published in Modern Science and Vedic Science. This related to a proof of Fermats Last Theorem using non traditional methods quiet unlike ~~those~~ the proof developed by Andrew Wyles. Although I did not read Wyles proof, few could unless they had in the region of thirty years experience in the rather esoteric mathematics required, I did read Singh's popular book on the topic.

In his paper Kapoor introduced the concept of real space geometry, a version of geometry which makes one very simple but profound change in the way a dimension is related to a domain. In standard Euclidean geometry the dimensions are always linear. Even for curvilinear geometries it is always assumed that the dimensions are one-dimensional. This makes the definition of a norm, or the definition of a measure, very simple using the standard Euclidean norm as the value of the square root of the sum of the squares of the distance from the prescribed origin. [needs to be stated more precicely].

For real space geometry however the

situation is quiet different due to the subtle change prescribed by S. K. Kapoor. This change is simple to describe but induces a level of complexity and challenge into the very basis of geometry that the logically trained mathematical mind of the Western mathematician really has to struggle to deal with it.

So what is this change, and what are its implications. The change is quiet simple and it states that there is a difference of order 2 between the dimensionality of a domain and the dimensionality of its associated dimension. In his original paper Kappor justifies his observation based on his research in Vedic Science and his interpretation of an aspect of Vedic literature, [give reference here]. This can be taken to be a similar process to the use of intuitionism in Western mathematics, as for example the discovery of ~~Fuschian~~ functions by Poincarre (?) or the discovery of the benzene ring by [?] which was based on a dream of a snake chasing it's tail [list possible sources for these stories including PI in the Sky, The Twelve Golden Problems? etc.] Perhaps one of the most famous dreams was that of George Boole and his counting of sheep [expand] which lead to his thesis "The Laws of Thought" and the development of modern mathematical logic.
[develop and talk about the grey stuff]

Notwithstanding the simplicity of the statement the notion that there is a difference of order 2 between the dimension of the domain and the dimension of the dimension has profound implications not only for the definition of a hypercube but as we shall shortly see for the structural content of a domain.

By way of expanding from the familiar territory of Euclidean geometry let us first see what are the comparisons between the structural components of a 3-Dimensional Euclidean domain E_3 and the structural components of a 3-Dimensional Real domain R_3.

In terms of linear elements the basic structural component of E_3 is a cube all the elements of which are 3-dimensional points including the surfaces, edges and corner points. No distinction is made between them. For real space geometry however there is a distinction. The inter elements (points) are three dimensional elements of R_3 the boundary surface elements are 2-Dimensional elements of R_2, the edge lines are 1-Dimensional elements of R_1 and the corner points are 0-Dimensional elements of R_0.

Simplyifing still further, if we

consider the representative body of 1-space if we consider the line segment. In Euclidean space, this is defined as a set of contiguous points each of 1-Dimension. For a 1-dimensional real space however there is a distinction as a closed line segment consists of 1-dimensional interior points with 0-dimensional boundaries.

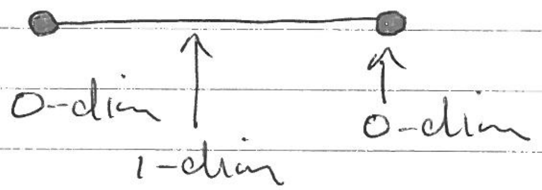


Fig 1.1

If a is the standard unit of length then the above geometrical formulation may be interpreted algebraically as

$$a^1 + 2a^0 \quad \text{Eq 1.1}$$

where the power relates to the dimension of the component. Thus the above algebraic formulation implies a single linear element a^1 with two zero elements placed appropriately at either end. In the real space formulation the notion of a sub element of the domain is different from the Euclidean formulation. Sub-elements of ~~real~~ R_1 are themselves 1-Dimensional and therefore have extension. I know from Kapoors other writings [Give Ref] that this has implications for such matters as Dedekind cuts etc.

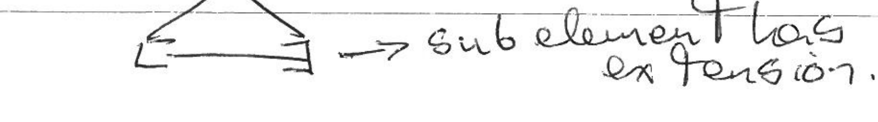
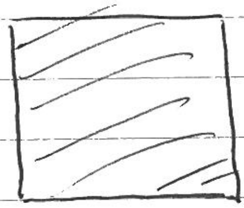


Fig 1.2

So what about R_2, real 2-space. Let us firstly just give the form of the representative body, the square in R_2, and then see how it may be generated from the representative body in R_1.

In 2-space a square consists of a 2-dimensional domain bounded by 4 lines ~~an~~ with 4 corner points.

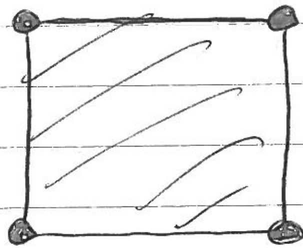
In E_2 this is represented simply as



[all one colour]

Fig 1.3

In R_2 however we distinguish between elements of different dimensionality by using colour codes.



Red – 0-dim
Green – 1-dim
Blue – 2-dim

Fig 1.4

These may be represented algebraically as

$$\underbrace{a^2}_{\text{Blue}} + \underbrace{4a^1}_{\text{Green}} + \underbrace{4a^0}_{\text{Red}} \quad \text{Eq 1.2}$$

← colour the lines

Comparing the algebraic formulations Eq 1.1 and Eq 1.2 we note the algebraic equivalence of Eq 1.2 with

$$(a^{\Phi} + 2a^{0}) \circ (a+2) \quad \text{Eq } 1.3$$

Comparing the geometrical forms Fig 1.1 and Fig 1.4 we note that the regular representative body in R_2 may be generated from the regular representative body in R_1 by moving each component along a track mathematically perpindicular to the line, or 1-space component. The distance moved is equal to the unit length a, but that is not the only process involved, in this generational process.

Firstly there is duplication, in that each structural component is replicated with one remaining fixed, possibly as a point of reference, and the second moving the requisit distance. Exactly why, and what are the implications of this replication are unclear at this stage.

So what does the movement produce.

A moving point produces two boundar points and a 1-dimensional body. Thus.

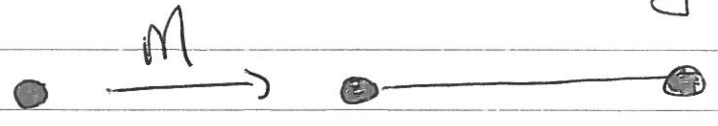


Fig 1.5

where M is a generational movement operator.

PTO

Similarly a moving line produces two lines and a 2-dimensional domain. Thus

[Note:- Use different colours for structural elements

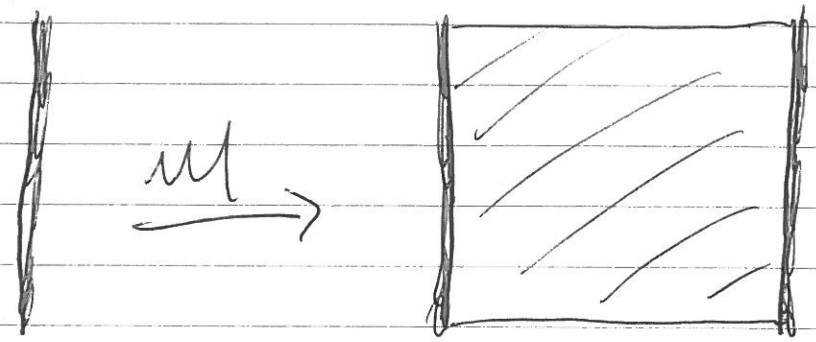


Fig 1.6

2-D region should be light blue

Keep all colours mute

In the above ~~to~~ movement transformation we do not include the boundary components of the 1-dim rob as they are handled separately.

Knittin all together, or considering the movements we get

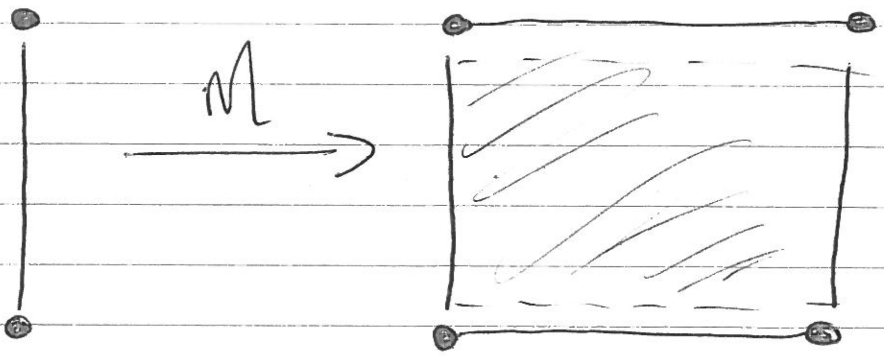


Fig 1.7

[Note:- This could easily be represented as a dynamic graphic.

P.T.O.

We can now interpret the algebraic formulation of Eq 1.3 as.

$$(a^1 + 2a^0) \underset{\substack{\uparrow \\ M}}{\bullet} (a + 2) \qquad \text{Eq } 1.4$$

where the multiplication or $\bullet$ operator is the algebraic equivalent of replication and movement through a distance corresponding to the unit length.

So where to now. Let us first chase the structural components of the cube, and then the hyper-cube in ~~4 - Real~~ R_4

If we look at the cube in E_3 we see that it consists of an interior domain ~~with~~ with six surface plates twelve edges and eight corner points. Again in E_3 no distinction is made between the dimensionality of ~~por~~ elements in the domain, on the surface, along the edges or at the corners.

Algebraically we get the following

$$(a^2 + 4a^1 + 4a^0) \bullet (a+2) \qquad \text{Eq } 1.5$$

$$\Rightarrow a^3 + 6a^2 + 12a^1 + 8a^0$$

which is the same as E_3 but with different components identified by their dimensional index

This is simply a form of binomial multiplicatio

Ex 1. Verify this representation by considering the movement of a rrp in R_2.

What is produced by moving the interior of the rrp in a mathematical direction perpindicular to the plane.

So simply put the cube in ~~3~~ R_3 is

$$C_3 = a^3 + 6a^2 + 12a^1 + 8a^0 \qquad \text{Eq } 1.6$$

What about the hyper-Cube C_4 in R_4?

Algebraically we may write

$$C_4 = C_3 \cdot (a+2) \qquad \text{Eq } 1.7$$

Giving the componentwise structure of C_4 as

$$C_4 = a^4 + 8a^3 + 24a^2 + 32a^1 + 16a^0$$

Eq 1.8

Ex 2 Verify Eq 1.8 by inducing the movement of each component in the geometrical form of ~~a~~ C_3 in a mathematically defined direction perpindicular to the 3-space ~~directions~~ axes

This gives the basic formulations of a sequence of regular representative bodies in $R_1 \to R_2 \to R_4 \to R_4$.

To summarise certain aspects we note that

- the dimensionality of the domain differs by an order of 2 from the dimensionality of the of the dimension.

- The following structural relationships exist for the cube

~~Boundary Domain~~

$$C_1 = a + 2a^0$$
$$C_2 = a^2 + 4a^1 + 4a^0$$
$$C_3 = a^3 + 6a^2 + 12a^1 + 8a^0$$
$$C_4 = a^4 + 8a^3 + 24a^2 + 32a^1 + 16a^0 \qquad \text{Eq 1.9}$$

- There is a domain boundary relationship of $(a+2)$.

Having considered the hypercube, we are now in a position to consider the hyper-sphere, and this is far from obvious. The reason for this is because there is no obvious method of formulating a norm, or unit of measure in R_4. This exposes what I call the hidden 'Axiom of Dimensionality' prevalent throughout all geometry derived from the Euclidean formulation. There is no reason why the dimensions of higher spaces should be linear. In fact setting them as linear induces in them a special

extraneous character which lies unexplained. Within the context of Real space geometry the dimensional spaces form a more naturally integrated component of the overall hierarchical structure. Their roles still need to be explicitly determined and they will change depending on the nature of the domain.

One point to note here is that odd spaces have dimensions of odd order and even spaces have dimensions of even order. This natural division into odd and even spaces suggests deep structural relationships between spaces.

So now the stage is set for us to begin considering how to chase the hyper-sphere in 4-space.

As I mentioned previously it is not possible to use the standard formulation of Euclidean spaces due to the non-availability of a norm in higher dimensional real spaces. The standard definition gives the surface of a sphere as the locus of all points equidistant from the central origin of the sphere. There appears to be a direct correspondence between the definition of a sphere in E_3 and R_3, since in R_3 we can use the Euclidean norm $\sqrt{x_1^2+x_2^2+x_3^2}$ however due to the planar dimensions in R_4 no such norm is available and we must proceed in a different way.

One possibility perhaps is to chase the hyper-sphere through a series of rotations. Although this method proved unsuccessful, it is worth noting it's principal features.

[may need more precision]

An important point at this juncture is to introduce the notation of topological equivalence. Two geometric bodies are topologically equivalent if they have the same degree of connectedness.

For instance the square and the circle are topoligically equivalent because any closed curve in either contains a region wholly contained within the body. The circle and annulus however are not.

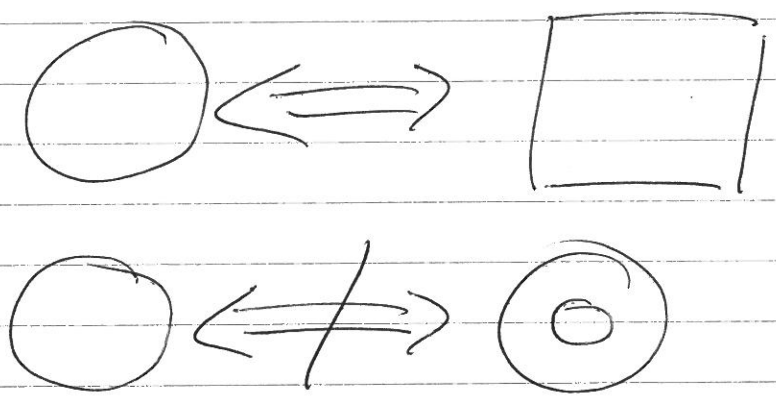


Fig 1.8

There is no continuous deformation which will allow us to transform two non topologically equivalent bodies.

<u>Note</u>:- In terms of continuous deformations it may be argued that the sharp corners in the square cannot be mapped to a circle, the famous squaring the circle problem. This in fact can be

handled computationally through the introduction of specialised corner functions similar to those used in the Edge Function Method ~~pio~~ pioneered by Paddy Quinlan in UCC.

In a sense topological equivalence holds the key to determining the structure of S_4 in R_4 the hyper sphere in Real ~~four~~ 4-space.

Let us first chase S_1, S_2, S_3 in R_1, R_2, R_3.

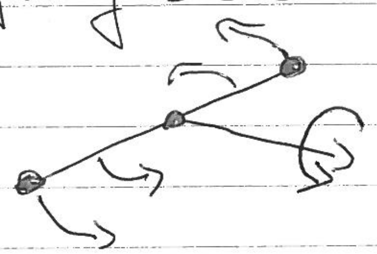
The sphere S_1 in R_1 is simply C_1, it has the same form as the cube possibly with an axis point defined at the centre.

Therefore S_1 has the form

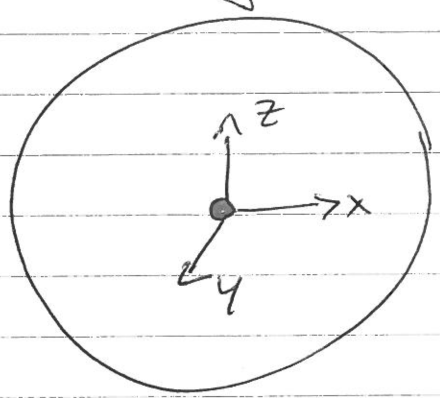
$S_1 \Rightarrow$ 

In his writings Sant Kapoor mentions the significance of the origin as a sealed point in the higher dimensional space, as a point of transcendence.

One transformation here is to rotate S_1 ~~through an~~ about an axis perpindicular to the origin of ~~the~~ S_1



This will induce a rotation in the structural components of S_1 to produce S_2

$S_2 \Rightarrow$ 

Note the point of transcendence at the origin.

A third rotation in ~~a plane perpendicular~~ along the z z-axis allows us to produce the sphere S_3. After this however things begin to get very complex as it is unclear what the structure of S_4 should be.

This is where a variation of topological equivalence may be used, to at least define some of the regions of S_4 and their forms.

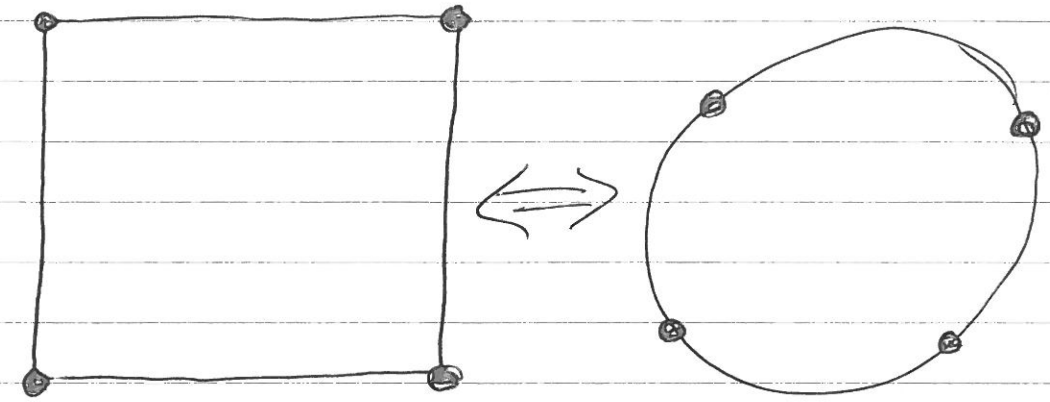
Let us chase this from S_1 to S_3, both pictorially and algebraically.
[3-D are in Legacy files]

§

$C_1 = a^1 + 2a^0$ $\qquad$ $S_1 = a^1 + 2a^0$

$C_2 = a^2 + 4a^1 + 4a^0$ $\qquad S_2 = a^2 + \underbrace{4a^1 + 4a^0}_{\text{regions of equivalence}}$



The equivalence here is that the structural components of the cube C_2 infer structural regions in the sphere S_2. Since the boundary of S_2 is continuous the cornerpoints do not actually exist, they are virtual points which can be used to describe regions of S_2.

In the case of S_3 and C_3 similar graphics can be (were) produced to illustrate the structural equivalence

Describe the research on wedge transformations in mathematica which was ultimately unsuccesful.

Note:- it is important to describe this, as ~~par~~ even false trails may lead to partial success.

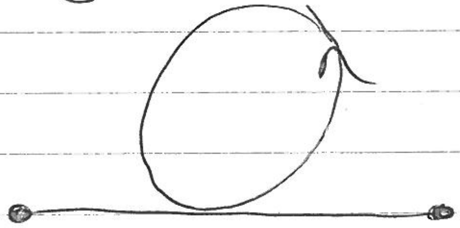
Interleaving Geometrical Bodies

One key intuition to the current approach to chasing S_4 in R_4 came while I was reviewing the book 'The Crowning Gem' by Kenneth Williams.

[Use full notes on this.]

I realised that there was a relationship between the algorithms he was presenting and the forms of the hyper-cubes.

I was considering how the one dimensional RRP refers back to itself in the creation of the 2-Dimensional RRP. I drew the following graphic



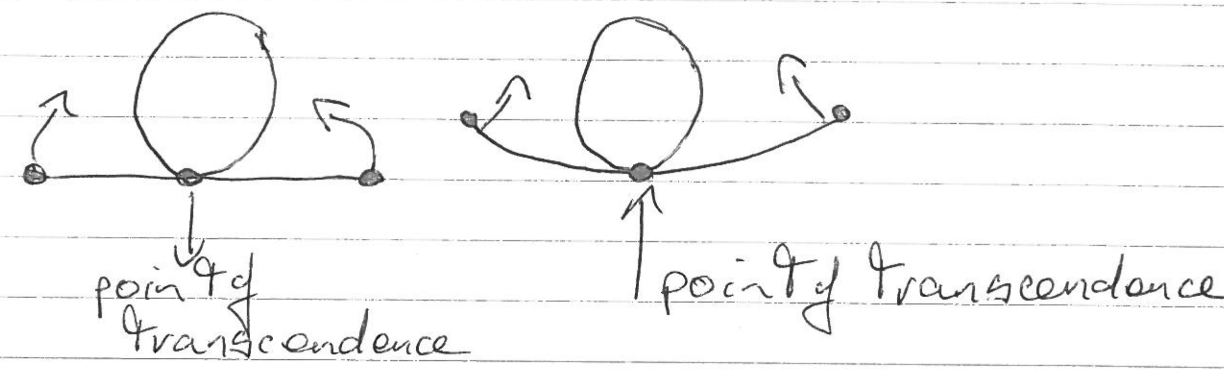
and recalled the Bhagavad Gita,
prakritim swam avasthabya
visrajami punah punah

"curving back on my own nature I create again and again."

Then I noted that by curving the line into a higher dimensional space

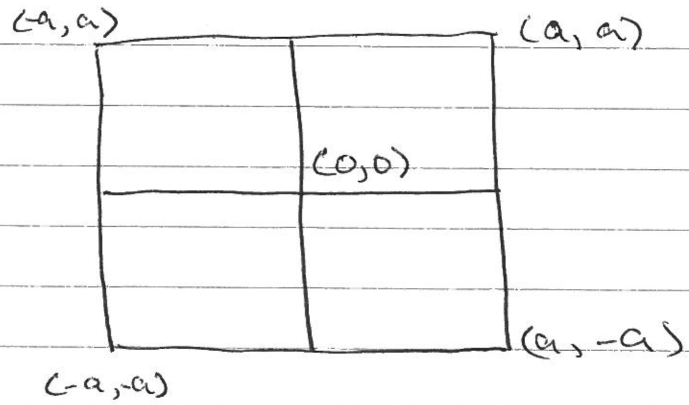
I could wrap the line about itself joining the end points to produce the boundary of a disk S_2.

~~I then~~



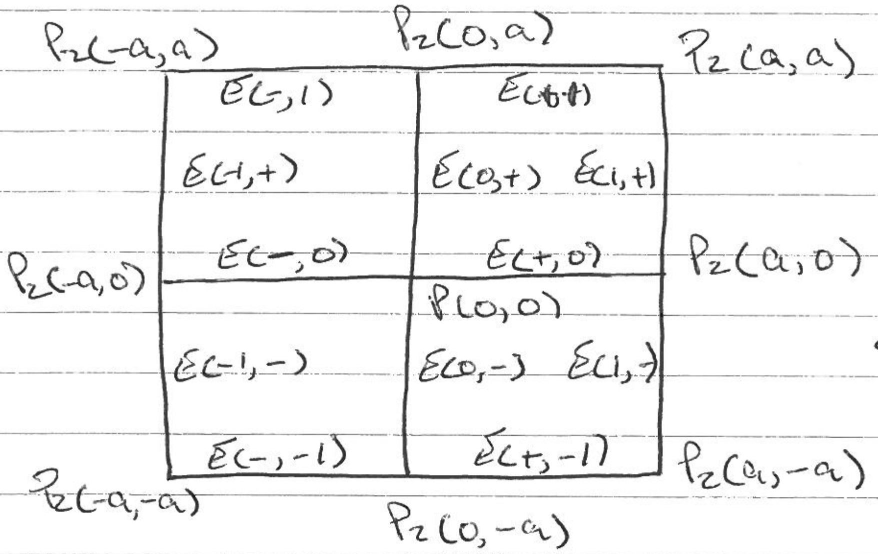
Next I considered how to do the same thing to wrap a square about the surface of a sphere. I called the transformation the four leaved clover, a symbol of luck in Ireland.

Basically consider a square of two units in length with corner points $P_2(a,a)$ $P_2(-a,a)$ $P_2(-a,-a)$ and $P_2(a,-a)$ ~~and~~ origin $P_2(0,0)$.



The computational steps required to wrap the square about the surface of the sphere may be outlined as follows.

Firstly the square is cut along it's axes and we identify various edges and interior points as follows



all the 1's should be a -

Note that in the notation for E a signifier of + indicates that the associated variable runs from 0 to +a and - indicates from -a to 0.

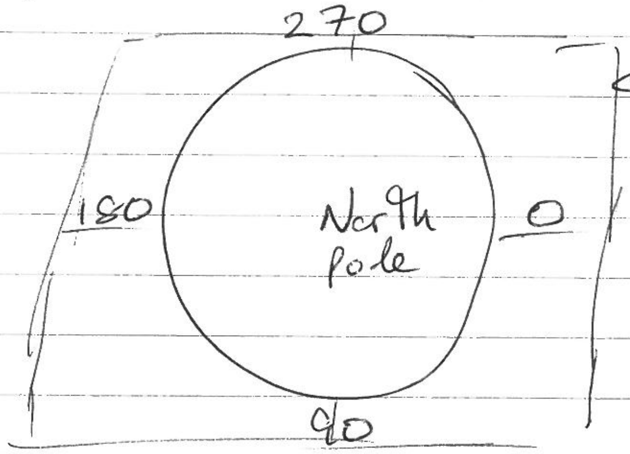
Thus the edge $E(a,+)$ signifies the edge corresponding to $x=a$, $0 \le y \le a$

With this in mind we place a sphere at the origin of the square. A sphere of 2a diameter, radius of a.

Considering initially the subsquare S_{++}, where $S_{++} = \{(x,y) : 0 \le x \le a, 0 \le y \le a\}$.

We effectively pull the point $P_2(a,a)$ until it sits on the northern pole of the sphere at $P_3(0,0,2a)$. Simultaneously we transform the point $P_2(0,a)$ until it coincides with $P_3(0,a,a)$ and we

straighten the edges $E(0+)$ and $E(+,a)$ so that they lie along a great circle passing through both poles (rem South pole is at origin of square) and is at a latitudinangle of 270°.



looking directly down on the sphere

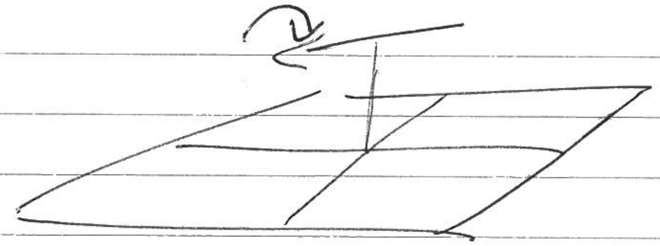
The edges $E(+,0)$ and $E(a,+)$ must be similarly straightened, Most likely this may be achieved using corner functions of a type mentioned previously.

There will also be deformations of the order of magnitude $\sqrt{2}a \to \pi a$ as the length of the diagonal of the subsquare is $\sqrt{2}a$ and the length of a great circle from North to south pole is πa.

When similar transformations are applied to each of the ~~four~~ three other quadrants the surface of the sphere is completely covered

An important point here is to note the Edges of the square which will be knitted together. They consist of $E(+,a)$ and $E(-,a)$ also $E(a,+)$ and $E(a,-)$ $E(-a,+)$ and $E(-a,-)$ also $E(-,-a)$ and $E(-,+a)$

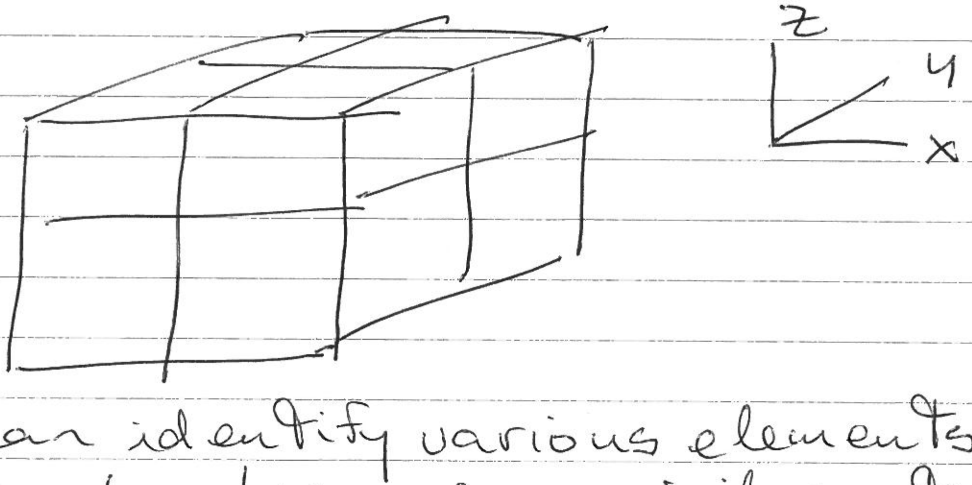
Note that the required transformation is not simply a rotation ~~is the~~ along the new axis, in this case out of the plane of the sphere but a combined rotation along the plane of the square with axis of rotation most likely located at the natural origin of the sphere. There is also stretching in two directions. The complete details need to be looked at mathematically. Note also that in this case four rotations are required, one for each quadrant.



Develop graphic for first quadrant.

This gives some idea as to how to proceed in order to chase S_4 in R_4.

In this case we take a cube and divide it into eight subcubes



We can identify various elements of the subcubes using a similar rotation as for the subsquares.

Thus for instance C_{+++} represents the first quadrant subcube on top.

It has external surface plates S_{++a}, S_{a++} and S_{+a+}, and internal surface plates S_{++0}, S_{0++} and S_{+0+}

It has ~~external~~ boundary edges E_{a+0}, E_{+a0}, E_{+0a}, E_{a+a}, E_{+aa}, E_{0+a}, E_{a0+}, E_{aa+}, E_{0a+} and so on.

What is needed here is to develop a simple program to completely enumerate all aspects of all components of the subcubes.

This can easily be done in any object oriented language.

The next step will be to induce rotations and transformations on each of the sub-cubes so that they generate the ~~surface~~ boundary of ~~a~~ the The hyper sphere S_4 in this case the ~~Surfa~~ boundary will be a continuous three dimensional volume possibly with certain dimensions disappearing as we traverse the boundary region from one structural region to another. ~~This~~ To determine the exact formulation of the transformations required will take a lot of work but we can get a clue as to what is required by examining the structure of C_4, the hypercube in R_4.

$$C_4 = a^4 + 8a^3 + 24a^2 + 32a^1 + 16a^0$$

Note that as defined, the four leaved clover maps the square onto four wedges or ~~sectors~~ segments of the sphere which have a different characterization to the structural equivalence between the sphere and the cube. It may be possible to develop the transformation in a manner more appropriate to the task at hand.

<u>Conclusion</u> :- This is a general outline of the computational steps required to develop S_4 in R_4. This task is worth doing as I feel that ultimately Real Space geometry holds a key to removing the singularities and other issues relating to the search for the Theory of Everything in modern Physics. My interest in considering this was piqued more than a decade ago after reading Brian Green's books on String Theory, particularly in relation to zero branes. These were zero dimensional mathematical objects which had structure. The only other place I had seen anything like this was in relation to R_0, zero dimensional real spaces which have dimensional components corresponding to R_{-2}

There is a whole wealth of possibilities to be developed here once the details of the transcending

transformations are clearly worked through.

Sin É

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